Abstract Algebra I
Exercise Sheet 5: Rings

Rings

Question 1. Decide whether each of the following is a ring. Determine which are fields. In the case the following is a ring, determine its additive and multiplicative identities:

- $(\mathbb{Z}, \cdot, +)$;
- $(\mathbb{Z}, \cdot, -)$;
- $(\mathbb{N}, \cdot, +)$;
- $(\mathbb{Q}, \cdot, +)$;
- $(\mathbb{R}, \cdot, +)$;
- $\{0, 2, 4, 6, 8\}$ with addition and multiplication modulo 10;
- $\{0, 3, 6, 9, 12, 15\}$ with addition and multiplication modulo 18;
- $(R[x], \cdot, +)$ for a ring $(R, +, \cdot)$ and some variable $x \notin R$;
- $(\mathbb{Z}, \cdot, +)$;

Question 2. Verify that $\mathbb{R}[i] = \mathbb{C}$ for $i^2 = -1$.

Homomorphisms

Question 3. Let 0_1 and 0_2 denote the additive identities of two rings R_1 and R_2. Verify that any map $\varphi : R_1 \to R_2$ satisfying $\varphi(x + y) = \varphi(x) + \varphi(y)$ for all $x, y \in R_1$ must have $0_2 \in \operatorname{Im}(\varphi)$ and $\varphi(0_1) = 0_2$.

Ideals

Question 4. Let R be a ring. Show that $\{0\}$ and R are both ideals in R.

Question 5. Show that $n\mathbb{Z}$ is an ideal in $\mathbb{Z}$ for all $n \in \mathbb{Z}$.